Fibonacci Melodies

Eduardo Ibargüengoytia

September 21, 2020

Music respresentation

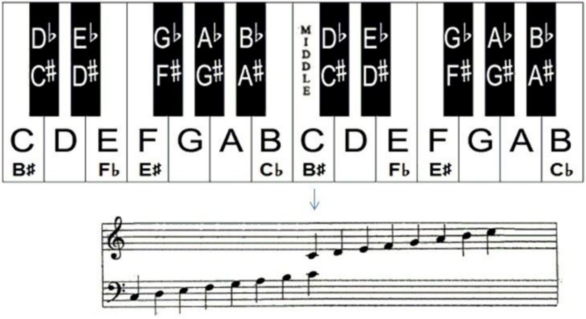


Figure: Piano and Score representation

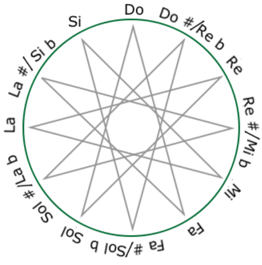


Figure: $\mathbb{Z}_{12}$ the 12 element cyclic group

1 — 2 — 3 — 4 — 5 — 6 — 7 — 8 — 9 — 10 — 11 — 12 — 13 — 14 — 15 — 16 — 17 — 18 — 19 — 20 — 21 — 22 — 23 — 24

↓

Preserves the addition operation. *At 11h my friend said we would meet in 3 hours, and I only have a wrist watch*. We call the operation Mod(12), and can be applied to arbitrary sets of integer numbers.

The **Fibonacci sequence** is a linear recursion defined by

$$f_{n+1} = f_{n-1} + f_n \quad \text{for } n \in \mathbb{N}_{\geq 1}$$

where f_n is the n-th Fibonacci number with $f_0 = 0$ and $f_1 = f_2 = 1$ This means that each number in the sequence is the sum of the two preceding ones. Starting with 0 and 1 as the first two terms of the sequence, the Fibonacci sequence looks like this for the first few terms :

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$$

Use of the sequence in visual arts and architecture.

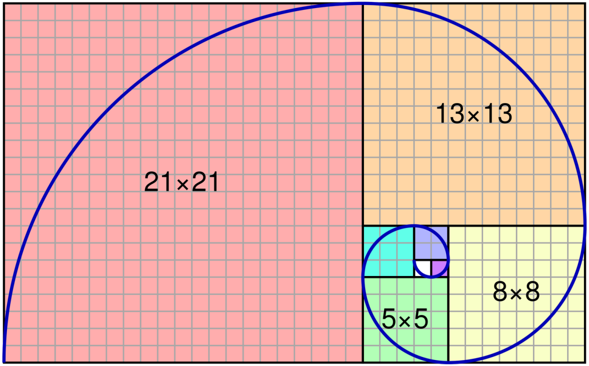


Figure: FIBONACCI SPIRAL

First Proposal

We use directly the operation Mod(12) to relate each fibonacci number to a unique note in the western music convention.
To each note we now associate a number:

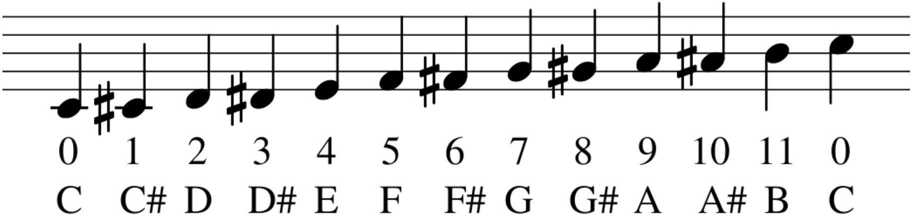


Figure: Full Scale

Explain the miracle!

Note	Fibonacci Sequence	Result
C	0	0
C#	1	1
C#	1	1
D	2	2
D#	3	3
F	5	5
G#	8	8
C#	13	1
A	21	9
A#	34	10
G	55	7
F	89	5
C	144	0
F	233	5
F	377	5
A#	610	10
D#	987	3
C#	1597	1
E	2584	4
F	4181	5
A	6765	9
D	10946	2
B	17711	11
C#	28657	1

Result

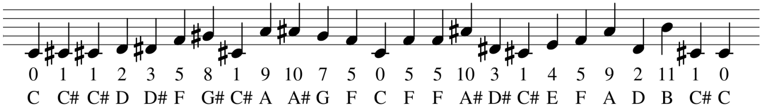


Figure: Full Scale - Mod(12)

Play

Other scales

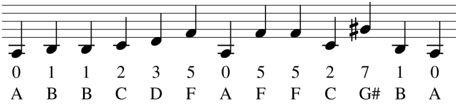
Harmonic A Minor Scale

The A Minor Scale consists of 8 notes:

$$A - B - C - D - E - F - G - G\#$$

We use the operation Mod(8) on the sequence, and get:

$$(0, 1, 1, 2, 3, 5, 0, 5, 5, 2, 7, 1)$$



Play

Play

Pentatonic scales

Play

Play