

Fibonacci Melodies

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Music representation

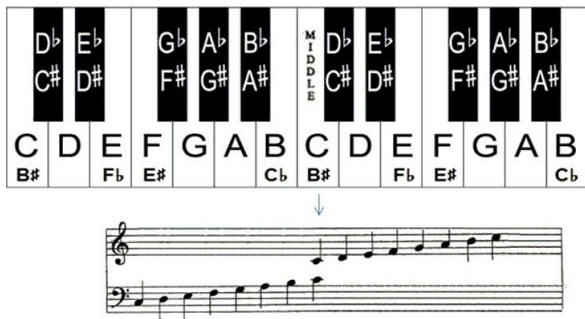


Figure: Piano and Score representation

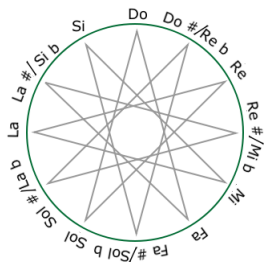


Figure: \mathbb{Z}_{12} the 12 element cyclic group

1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 10 - 11 - 12 - 13 - 14 - 15 - 16 - 17 - 18 - 19 - 20 - 21 - 22 - 23 - 24



Preserves the addition operation. *At 11h my friend said we would meet in 3 hours, and I only have a wrist watch.* We call the operation $\text{Mod}(12)$, and can be applied to arbitrary sets of integer numbers.

The **Fibonacci sequence** is a linear recursion defined by

$$f_{n+1} = f_{n-1} + f_n \quad \text{for } n \in \mathbb{N}_{\geq 1}$$

where f_n is the n -th Fibonacci number with $f_0 = 0$ and $f_1 = f_2 = 1$. This means that each number in the sequence is the sum of the two preceding ones. Starting with 0 and 1 as the first two terms of the sequence, the Fibonacci sequence looks like this for the first few terms :

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

Use of the sequence in visual arts and architecture.

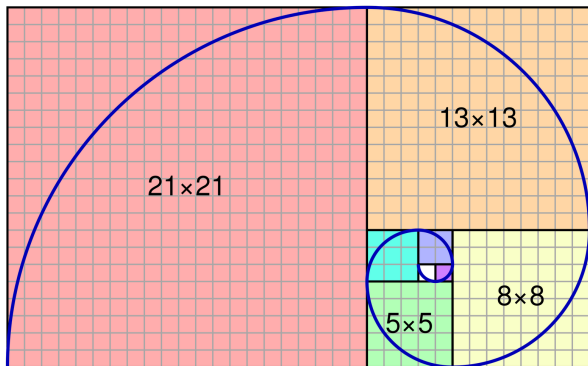


Figure: FIBONACCI SPIRAL

First Proposal

We use directly the operation $\text{Mod}(12)$ to relate each fibonacci number to a unique note in the western music convention.

To each note we now associate a number:

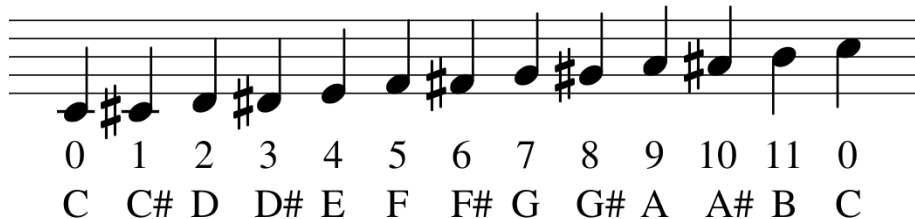


Figure: Full Scale

Explain the miracle!

Note	Fibonacci Sequence	Result
C	0	0
C#	1	1
C#	1	1
D	2	2
D#	3	3
F	5	5
G#	8	8
C#	13	1
A	21	9
A#	34	10
G	55	7
F	89	5
C	144	0
F	233	5
F	377	5
A#	610	10
D#	987	3
C#	1597	1
E	2584	4
F	4181	5
A	6765	9
D	10946	2
B	17711	11
C#	28657	1

Result



A musical staff showing a sequence of 12 notes. Below the staff are two rows of text: the first row contains numbers representing fingerings (0-11), and the second row contains the corresponding note names (C, C#, C#, D, D#, F, G#, C#, A, A#, G, F, C, F, F, A#, D#, C#, E, F, A, D, B, C#, C).

0	1	1	2	3	5	8	1	9	10	7	5	0	5	5	10	3	1	4	5	9	2	11	1	0
C	C#	C#	D	D#	F	G#	C#	A	A#	G	F	C	F	F	A#	D#	C#	E	F	A	D	B	C#	C

Figure: Full Scale - Mod(12)

Play

Other scales

Harmonic A Minor Scale

The A Minor Scale consists of 8 notes:

A – B – C – D – E – F – G – G#

We use the operation Mod(8) on the sequence, and get:

(0, 1, 1, 2, 3, 5, 0, 5, 5, 2, 7, 1)

Musical notation for the A Minor scale on a single staff. The notes are A, B, B, C, D, F, A, F, F, C, G#, B, A. Below the notes are the fingerings: 0, 1, 1, 2, 3, 5, 0, 5, 5, 2, 7, 1, 0. Below the fingerings are the note names: A, B, B, C, D, F, A, F, F, C, G#, B, A.

Play

Musical notation for the A Minor scale in 6/8 time, showing the sequence of notes: A, B, B, C, D, F, A, F, F, C, G#, B, A.

Play

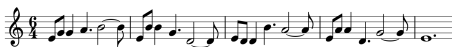
Pentatonic scales



A guitar fretboard diagram showing a pentatonic scale across two strings. The notes are: E (open), G (1), A (1), B (2), E (3), B (3), B (3), G (1), D (4), E (0), D (4), D (4), B (3), A (2), E (0), A (2), A (2), D (4), G (1), E (0).

0 1 1 2 3 0 3 3 1 4 0 4 4 3 2 0 2 2 4 1 0
E G A B E B B G D E D D B A E A A D G E

Play



Musical notation for a pentatonic scale in 4/4 time, starting on E4. The notes are: E4 (quarter), G4 (quarter), A4 (quarter), B4 (quarter), E5 (half), B4 (quarter), B4 (quarter), G4 (quarter), D4 (quarter), E4 (quarter), D4 (quarter), D4 (quarter), B4 (quarter), A4 (quarter), E4 (quarter), A4 (quarter), A4 (quarter), D4 (quarter), G4 (quarter), E4 (half).

Play